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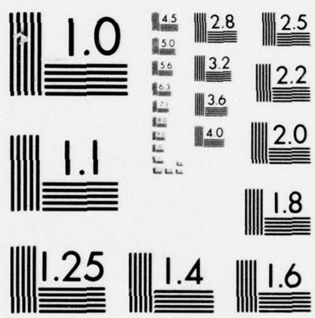

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AN APPROXIMATE ANALYSIS OF A MULTI-SERVER FINITE QUEUE WITH HETEROGENEOUS CUSTOMERS

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ABSTRACT

This paper presents a simple approach for the study of the following two types of multiserver finite queueing systems each with two classes of customers demanding service with different arrival and service rates: (1) Customers belonging to one class have priority over the other and the number of low priority customers that may be present in the system at any time is restricted. Customers blocked from entering service at any time are cleared from the system. (2) Customers belonging to one class are cleared when blocked (loss system) and customers belonging to the other class are buffered (delay system) under such circumstances. Furthermore, the number of customers belonging to the loss system demanding service at any time is restricted.

INTRODUCTION

Consider the following two classes of multi-server finite queueing systems:

1) Multi-server loss systems with two classes of customers with different arrival and service rates. Customers belonging to one class have priority over the other and the number of low priority customers that may be present in the system at any time is restricted by a number K ($K \leq s$, number of servers). These systems will be denoted as $A/B_1,B_2/s/s$.

2) Multi-server systems with two classes of customers in which customers belonging to one class are cleared when blocked (loss system), and customers belonging to the second class are buffered under such circumstances (delay system). The two classes have different arrival and service rates. As before, the number of customers belonging to the loss system that may demand service at any time is restricted by K. We denote this class of systems by $A/B_1,B_2/s/N$.

Such queues are common in the study of telecommunications networks in which several classes (such as voice and data) share access and traffic line facilities. Under purely Markovian assumptions, the simplest of such systems have been analyzed by Cohen [2]. However, when the number of servers is greater than one, the magnitude of the problem increases considerably as illustrated in Kotiah and Slater [3], where a two server system of this type has been analyzed. For larger values of s, even though an exact analysis is possible in principle, in practice there are severe computational limitations (as also pointed out by Bhat and Fischer [1]). An approximate technique to determine

some of the system performance measures has been presented in Bhat and Fischer.

Here we are concerned with the steady state marginal distributions of each class from which one could easily deduce the expected values and variances of number of customers and the respective blocking probabilities. Bearing in mind the utility of the derived results and keeping the generality of the above description to a large extent, we shall assume the following common characteristics for the two classes of multi-server systems mentioned above:

(1) The two classes of customers arrive in a single recurrent process with mean $(1/\lambda)$. An arriving customer belongs to one of the two classes with constant probabilities p and 1-p respectively.

(2) Both classes of customers have exponential service times with rates μ_1 and μ_2 respectively.

(3) There are s servers in parallel and if buffering is allowed, no more than n customers are allowed in the system at one time.

(4) An arriving class 2 (low priority) customer can not enter the system if K of them are receiving service already and/or the buffer has some class 1 customers waiting.

With these simplifications, the two classes of queueing systems may now be represented as $GI/M_1,M_2/s/s$ and $GI/M_1,M_2/s/N$ respectively. In the above description of the problem, the queue-length process, when restricted to the arrival epochs, describes a discrete time Markov chain. Clearly, the measures of interest can be easily deduced, if we obtain the stationary distribution of this imbedded Markov chain. However, in reality, for moderate to large systems, the number of states

$[(K+1)(N+1-K/2)^{+}$ for $GI/M_1,M_2/s/N$, $N \geq s]$ is going to be too large to make an exact numerical approach for the determination of the stationary distribution of the Markov chain possible. Furthermore, because of the cumbersome nature of the expressions involving binomial coefficients for the one step transition probabilities of the Markov chain, generation of the transition probability matrix (TPM) itself might pose considerable numerical problems. This will be exemplified in the following section. Therefore, alternate techniques are needed to provide at least approximate values for system performance measures.

In this paper we develop approximation techniques for determining the first two moments of system sizes and the probabilities of blocking for each of the customer classes. Exact analysis is used to ascertain the adequacy of our heuristic procedure. Based on our computational experience, the approximation provides better results for the system $GI/M_1,M_2/s/s$ than for the system $GI/M_1,M_2/s/N$. Nevertheless, in the absence of an analytical justification these conclusions are restricted to the class of systems used in the analysis. Incidentally, since the expressions for the one step transition probabilities in the case of $GI/M_1,M_2/s/N$ are very complicated for numerical work, a justification for the procedure should come from an elaborate simulation study of these systems, which in itself is a major problem.

In the systems of interest, if $\mu_1 = \mu_2 = \mu$, numerical methods developed in Raju [4] and Raju and Bhat [5] would directly yield the measures of interest since there is no need to distinguish the customers when once they join the system. In an indirect way, the procedure used in this paper heavily utilizes this situation. For

further development we introduce the following notations:

$A(x)$: Inter-arrival time distribution ($0 \leq x < \infty$)

ρ_i : Traffic intensity of class i ($i = 1, 2$) customers ($= \lambda p/\mu_1$ for $i=1$ and $= \lambda(1-p)/\mu_2$ for $i=2$).

Q_n^i : Number of class i customers at the n^{th} arrival epoch in the system.

Q_n : Number of customers in the system at the n^{th} arrival epoch ($= Q_n^1 + Q_n^2$).

Define,

$$\alpha_{ij;rt} = P\{Q_{n+1}^1 = r,\ Q_{n+1}^2 = t \mid Q_n^1 = i,\ Q_n^2 = j\}$$

Thus $\alpha_{ij;rt}$ represents the transition probability of the process (Q_n^1, Q_n^2) from $i \to r$ and $j \to t$ respectively during an interarrival time period. The TPM of the imbedded Markov chain has α's for its elements. With this background, we shall next consider the approximations for the respective systems.

APPROXIMATION FOR $GI/M_1,M_2/s/s$ SYSTEM:

Recall that an arriving customer is of class 1 with probability p and of class 2 with probability (1-p). For the sake of simplicity, we let

$$f(m) = 1 - e^{-m\mu_1 x}$$

$$g(n) = 1 - e^{-n\mu_2 x}$$

and

$$h(m,n) = e^{-(m\mu_1 + n\mu_2)x} \qquad (0 \leq x < \infty)$$

We have for this system

$$\alpha_{ij;rt} = p\binom{i+1}{r}\binom{j}{t}\int_0^\infty f(i+1-r)\ g(j-t)\ h(r,t)\ dA(x)$$

$$+ (1-p)\binom{i}{r}\binom{j+1}{t}\int_0^\infty f(i-r)\ g(j+1-t)\ h(r,t)\ dA(x)$$

$$(0 \le i + j < s,\ 0 \le j < K,\ 0 \le r \le i + 1,\ 0 \le t \le j + 1)$$

$$\alpha_{s-j,j;rt} = \binom{s-j}{r}\binom{j}{t}\int_0^\infty f(s-j-r)\ g(j-t)\ h(r,t)\ dA(x)$$

$$(j \le K,\ r \le s - j,\ t \le j)$$

and

$$\alpha_{iK;rt} = p\binom{i+1}{r}\binom{K}{t}\int_0^\infty f(i+1-r)\ g(K-t)\ h(r,t)\ dA(x)$$

$$+ (1-p)\binom{i}{r}\binom{K}{t}\int^\infty f(i-r)\ g(K-t)\ h(r,t)\ dA(x)$$

$$(i + j < s,\ r \le i + 1,\ t \le K)$$

The cumbersome nature of the expressions is evident and moreover, it does not seem possible to get around the problem of direct evaluation of binomial coefficients. Thus, even for moderate values of s and/or K, the generation of TPM might turn out to be expensive and unreliable. However, if $\mu_1 = \mu_2 = \mu$, it is clear that $\{Q_n : n = 1, 2, 3, \ldots\}$ is a Markov chain imbedded in the queue-length process. Additionally assuming K = s in the above system, we obtain a system of the type GI/M/s/s. Let

$$P_{ij} = P\ \{Q_{n+1} = j \mid Q_n = i\} \qquad (0 \le i,\ j \le s).$$

we have

$$P_{ij} = \begin{cases} p\binom{i+1}{j}\displaystyle\int_0^\infty (1-e^{-\mu x})^{i+1-j}\ e^{-j\mu x}\ dA(x) \\ \quad + (1-p)\binom{i}{j}\displaystyle\int_0^\infty (1-e^{-\mu x})^{i-j}\ e^{-j\mu x}\ dA(x) \\ \qquad\qquad\qquad\qquad (i < s,\ j \le i + 1) \\ \binom{s}{j}\displaystyle\int_0^\infty (1-e^{-\mu x})^{s-j}\ e^{-j\mu x}\ dA(x) \qquad (j \le s,\ i = s) \end{cases} \tag{1}$$

The resulting matrix $P \equiv \| P_{ij} \|$ is almost left triangular (or lower Hessenberg) and can be easily generated. Denote

$$\alpha_j = \int_0^\infty e^{-j\mu x}\, dA(x) \qquad (0 \leq j \leq s)$$

After some algebraic manipulation, we can establish that P can be generated in two stages through the following recursive relations.

<u>First Stage</u>

$$P_{i,i+1} = p\alpha_{i+1}$$

$$P_{ii} = p(i+1)[\alpha_i - \alpha_{i+1}] \qquad (0 \leq i < s)$$

$$P_{ij} = \frac{i+1}{i-j+1} P_{i-1,j} - \frac{j+1}{i-j+1} P_{ij+1} \qquad (1 \leq i < s,\ j < i)$$

and

$$P_{sj} = P_{s-1j}/p \qquad (0 \leq j \leq s)$$

<u>Second Stage</u>

Starting from $i = s - 1, s - 2, \ldots, 1$ and for $j < i$

$$P_{ij} = P_{ij} + \frac{1-p}{p} P_{i-1j}$$

Then reset the diagonal entries to

$$P_{ii} = P_{ii} + (1-p)\,\alpha_i \qquad (0 \leq i < s)$$

Remark: It may be noted that if we set p=1 in 1 , we get the expressions for the ordinary GI/M/s/s system. Using the first stage of the above recurrence relations with p=1, we can very efficiently generate the TPM for this system.

The objective is to approximate the measures of interest for the

system $GI/M_1,M_2/s/s$ by the corresponding measures of two different systems of the type GI/M/s/s for which the characteristics can be easily determined by the methods developed in Raju [4]. For the sake of clarity, we shall denote this approximating system as GI*/M/s/s. Given below is the approximation scheme for $GI/M_1,M_2/s/s$ system.

<u>The Heuristic</u>:

Initialize:

$$N_2 = [K \cdot \rho_2/(\rho_1 + \rho_2)]^+$$

$$N_1 = s - N_2$$

Step 1:

Solve $GI^*/M_1/N_1/N_1$ (i.e., in (1), put $\mu = \mu_1$, $s = N_1$)

obtain $N_1^* = E[\text{Busy Servers}]$ in the above system

Set $N_2 = [s - N_1^*]^+$

<u>Restart</u>: $N_2 = \min(N_2, K)$

Step 2:

Solve $GI^*/M_2/N_2/N_2$ (in (1), put $\mu = \mu_2$, $s = N_2$ and interchange p and 1-p)

Obtain the stationary distribution $\{\pi_2 = \pi_{2j} : 0 \leq j \leq N_2\}$ of the imbedded Markov chain. Let E_2 and V_2 be the expected value and variance of the distribution π_2 respectively.

Set $N_1 = (s-E_2)^+$

Step 3:

Solve $GI^*/M_1/N_1/N_1$ (in (1), put $\mu = \mu_1$, $s = N_1$)

Obtain the stationary distribution $\{\pi_1 = \pi_{1j} : 0 \leq j \leq N_1\}$ of the imbedded Markov chain. Let E_1 and V_1 be the expected

value and variance of the distribution π_i respectively.

Step 4:

Stop (or go to Restart with $N_2 = (s-E_1)^+$).

The measures of interest for the system $GI/M_1,M_2/s/s$ are now approximated by E_i, V_i and π_{iN_i} which represent for class i (i=1,2), the expected number of customers, variance of the number of customers and probability of blocking respectively.

Remark: If one wishes to improve upon the results obtained after the first iteration, one could go to Restart from step 4. However, we do not have enough evidence to suggest that such an attempt would always yield better results.

APPROXIMATION FOR $GI/M_1,M_2/s/N$ SYSTEM

The basic idea behind our procedure is to approximate the characteristics of interest for the system $GI/M_1,M_2/s/N$ by the corresponding measures of two different systems of the type GI/M/s/N for which the measures can be readily obtained by the methods developed in Raju [4]. For the sake of clarity we shall denote the approximating system as $GI^*/M/s/N$.

The Heuristic:

Initialize:

$$s_2 = [K \cdot \rho_2/(\rho_1 + \rho_2)]^+ , \; N_2 = K$$

Step 1:

Solve $GI^*/M_2/s_2/N_2$,

obtain: $s_1^* = E$ [number of customers in the system]

Step 2:

Solve $GI^*/M_1/s_1/N_1$, where

$$s_1 = s - s_1^*, \ N_1 = N - s_1^* .$$

Obtain the stationary distribution $\{\pi_{1j}, \ 0 \leq j \leq N_1\}$ of the imbedded Markov chain, hence get E_1, V_1, the expected value and variance of the stationary distribution respectively.

Obtain $s_2^* = E$ [busy servers] due to the above system.

Step 3:

Solve $GI^*/M_2/s_2/N_2$, where

$$s_2 = N_2 = [\min [K, s - s_2^*]]^+.$$

Obtain the stationary distribution $\{\pi_{2j}, \ 0 \leq j \leq N_2\}$ of the imbedded Markov chain. Let E_2, V_2 be the expected value and variance of this distribution respectively.

Step 4: STOP.

The measures of interest for the system $GI/M_1,M_2/s/N$ are now approximated by E_i, V_i and π_{iN_i} which represent for class i(i=1, 2) the expected number of customers, variance of the number of customers and probability of blocking respectively.

NUMERICAL ILLUSTRATIONS:

A. The System $GI/M_1,M_2/s/s$

For the purpose of illustrating the approximation developed for the system $GI/M_1,M_2/s/s$ in the preceding section, we present below tables containing exact measures of performance for the system and their respective approximations obtained through our heuristic under special cases. It is assumed that the time intervals between successive arrival epochs have an Erlangian distribution (Gamma distribution) with a probability density function

$$f(x) = e^{-m\lambda x} \frac{(m\lambda)^m \ x^{m-1}}{(m-1)!} dx \qquad m=1, 2, 3, \ldots; \ \lambda > 0 .$$

The arrival rate is given by λ. The following notations are used.

System parameters:

s : system capacity and the number of servers.

K : system capacity for class 2 access

p : probability of class 1 arrival

λ : arrival rate

m : number of phases in the Erlangian distribution

c_v : coefficient of variation of arrival process

: $1.0/\sqrt{m}$

μ_1 : service rate for class 1 customers

μ_2 : service rate for class 2 customers

Measures of performance:

$E(Q_i)$: mean number of class $i(=1$ or $2)$ customers at arrival epochs.

$V(Q_i)$: variance of number of class $i(=1$ or $2)$ customers in system at arrival epochs

PB_i : probability of blocking for class i $(i=1, 2)$.

Approximating systems parameters:

N_i: system capacity and the number of servers for class i $(=1, 2)$

λ_i: arrival rate for class i $(=1, 2)$ $= p\lambda$ if $i=1$, $= (1-p)\lambda$ if $i=2$.

The interarrival times for class i $(=1, 2)$ system are assumed to have an Erlangian distribution with probability density function

$$f_i(x) = e^{-m\lambda_i x} \frac{(m\lambda_i)^m x^{m-1}}{(m-1)!} dx \qquad \lambda_i > 0$$

The measures of performance of the system $GI/M_1,M_2/s/s$ are approximated by the measures of performance of $GI^*/M_1/N_1/N_1$ and $GI^*/M_2/N_2/N_2$ systems. In Tables 1a and 1b, for each set of variables the exact measures for the system $GI/M_1,M_2/s/s$ are presented and the corresponding approximations obtained from the approximating systems appear immediately below. For instance, the digits inside the braces following the number of servers in the system $GI/M_1,M_2/s/s$ represent N_1 and N_2 respectively.

Based on our numerical work the following observations can be made.

1. The approximation performs well under low traffic conditions which are combinations of the arrival rate, service rate and system capacity.
2. The effect of the approximation is more pronounced on the variance of the number of customers in the system.
3. The effect of the approximation is more pronounced under higher variability conditions of the arrival process.

B. The System $GI/M_1,M_2/s/N$

Because of the analytically intractable nature of the one-step transition probability expressions (obtained by extending the expressions given in (1) for cases $s \leq i,j \leq N$), the exact analysis of this system is extremely cumbersome and time consuming. However, if we assume exponential inter-arrival times for each of the two classes of customers, the desired characteristics of the above system can be obtained by solving the balance of state equations. (See, Bhat and Fischer [1]). Therefore, numerical comparisons for this system approximating procedure shall be restricted to Poisson arrivals.

G / M1,M2 / S / S SYSTEM

S	K	LAM	U1	U2	COV	E(Q1)	E(Q2)	V(Q1)	V(Q2)	PB1	PB2
12	4	6.000	2.000	.400	.354	1.04	3.49	.89	.51	.0000	.3607
(9, 4)	-					1.04	3.49	.89	.51	.0000	.3607
12	4	6.000	2.000	.400	.447	1.05	3.49	.91	.52	.0000	.3614
(9, 4)	-					1.05	3.49	.91	.52	.0000	.3614
12	4	6.000	2.000	.400	.707	1.11	3.48	1.02	.54	.0000	.3641
(9, 4)	-					1.11	3.48	1.02	.54	.0000	.3641
12	4	6.000	2.000	.400	1.000	1.20	3.48	1.20	.58	.0000	.3683
(9, 4)	-					1.20	3.48	1.20	.58	.0000	.3683
12	4	6.000	2.000	.400	1.545	1.38	3.47	1.49	.65	.0000	.3778
(9, 4)	-					1.38	3.47	1.49	.65	.0000	.3778
12	4	6.000	2.000	.400	2.047	1.54	3.48	1.73	.69	.0001	.3875
(9, 4)	-					1.54	3.48	1.73	.69	.0000	.3875
12	4	12.000	2.000	.400	.354	2.23	3.76	1.87	.24	.0001	.4728
(9, 4)	-					2.23	3.76	1.87	.24	.0000	.4728
12	4	12.000	2.000	.400	.447	2.24	3.76	1.92	.25	.0002	.4730
(9, 4)	-					2.25	3.76	1.92	.25	.0000	.4730
12	4	12.000	2.000	.400	.707	2.30	3.76	2.09	.25	.0003	.4737
(9, 4)	-					2.30	3.76	2.10	.25	.0001	.4737
12	4	12.000	2.000	.400	1.000	2.40	3.75	2.37	.27	.0008	.4748
(9, 4)	-					2.40	3.76	2.39	.27	.0003	.4748
12	4	12.000	2.000	.400	1.545	2.60	3.75	2.88	.29	.0022	.4777
(9, 4)	-					2.61	3.75	2.94	.29	.0008	.4777
12	4	12.000	2.000	.400	2.047	2.82	3.75	3.32	.32	.0040	.4810
(9, 4)	-					2.83	3.75	3.43	.32	.0016	.4809
12	4	18.000	2.000	.400	.354	3.40	3.84	2.73	.16	.0030	.5138
(9, 4)	-					3.42	3.84	2.81	.16	.0010	.5137
12	4	18.000	2.000	.400	.447	3.42	3.84	2.78	.16	.0033	.5138
(9, 4)	-					3.43	3.84	2.87	.16	.0012	.5138
12	4	18.000	2.000	.400	.707	3.46	3.84	2.97	.16	.0046	.5141
(9, 4)	-					3.49	3.84	3.08	.16	.0018	.5141
12	4	18.000	2.000	.400	1.000	3.54	3.84	3.25	.17	.0070	.5147
(9, 4)	-					3.57	3.84	3.42	.17	.0031	.5146
12	4	18.000	2.000	.400	1.545	3.72	3.84	3.79	.18	.0124	.5160
(9, 4)	-					3.77	3.84	4.08	.18	.0062	.5159
12	4	18.000	2.000	.400	2.047	3.91	3.83	4.24	.20	.0183	.5176
(9, 4)	-					3.99	3.84	4.66	.20	.0100	.5175

G / M1,M2 / S / S SYSTEM

S	K	LAM	U1	U2	COV	E(Q1)	E(Q2)	V(Q1)	V(Q2)	PR1	PR2
5	2	3.000	1.000	6.250	.354	1.03	.11	.88	.10	.0007	.0024
(5, 2)	-					1.03	.11	.88	.10	.0005	.0014
5	2	3.000	1.000	6.250	.447	1.05	.13	.90	.12	.0009	.0034
(5, 2)	-					1.05	.13	.91	.12	.0006	.0022
5	2	3.000	1.000	6.250	.707	1.10	.19	1.00	.17	.0021	.0096
(5, 2)	-					1.10	.19	1.01	.18	.0012	.0069
5	2	3.000	1.000	6.250	1.000	1.18	.28	1.14	.26	.0052	.0244
(5, 2)	-					1.19	.28	1.16	.26	.0025	.0187
5	2	3.000	1.000	6.250	1.545	1.34	.35	1.34	.32	.0106	.0412
(5, 2)	-					1.36	.36	1.40	.33	.0052	.0304
5	2	3.000	1.000	6.250	2.047	1.48	.40	1.49	.36	.0158	.0538
(5, 2)	-					1.51	.41	1.58	.37	.0081	.0381
5	2	6.000	1.000	6.250	.354	2.11	.33	1.53	.28	.0218	.0472
(5, 2)	-					2.15	.35	1.59	.29	.0151	.0191
5	2	6.000	1.000	6.250	.447	2.11	.35	1.54	.29	.0236	.0518
(5, 2)	-					2.16	.37	1.62	.31	.0160	.0223
5	2	6.000	1.000	6.250	.707	2.13	.40	1.59	.34	.0305	.0697
(5, 2)	-					2.19	.43	1.71	.36	.0194	.0354
5	2	6.000	1.000	6.250	1.000	2.15	.48	1.65	.41	.0414	.0972
(5, 2)	-					2.25	.52	1.84	.44	.0250	.0571
5	2	6.000	1.000	6.250	1.545	2.24	.57	1.78	.46	.0573	.1311
(5, 2)	-					2.38	.63	2.05	.50	.0355	.0814
5	2	6.000	1.000	6.250	2.047	2.34	.62	1.86	.49	.0705	.1559
(5, 2)	-					2.52	.70	2.19	.54	.0455	.0978
5	2	9.000	1.000	6.250	.354	2.82	.49	1.54	.38	.0715	.1343
(5, 2)	-					2.97	.58	1.68	.43	.0530	.0546
5	2	9.000	1.000	6.250	.447	2.81	.50	1.54	.39	.0736	.1390
(5, 2)	-					2.98	.59	1.69	.44	.0541	.0590
5	2	9.000	1.000	6.250	.707	2.79	.54	1.57	.43	.0815	.1561
(5, 2)	-					2.99	.64	1.75	.48	.0585	.0754
5	2	9.000	1.000	6.250	1.000	2.76	.60	1.60	.47	.0928	.1804
(5, 2)	-					3.01	.72	1.85	.54	.0652	.1001
5	2	9.000	1.000	6.250	1.545	2.79	.68	1.68	.51	.1099	.2139
(5, 2)	-					3.08	.83	2.00	.58	.0784	.1305
5	2	9.000	1.000	6.250	2.047	2.83	.73	1.74	.53	.1241	.2391
(5, 2)	-					3.17	.90	2.10	.60	.0907	.1517

Tables 2a and 2b present exact measures of performance and their respective approximations obtained through our heuristic procedure. The following notations are used:

Systems Parameters

N : System capacity

s : Number of servers

K : System capacity for class 2 access

λ_i : Arrival rate of class i(=1, 2) customers

μ_i : Service rate of class i(=1, 2) customers

Measures of Performance:

$E(Q_i)$: Mean number of class i(=1, 2) customers in the system.

$V(Q_i)$: Variance of class i(=1, 2) customers in the system.

PB_i : Probability of blocking for class i(=1, 2).

For each set of system parameters, the approximate measures of performance obtained through our heuristic are presented right below the corresponding exact measures.

A general observation that can be made is that the approximation gives poorer results under heavier traffic conditions. Also for reasons which need further investigations, the performance of the heuristic procedure on this class of systems is poorer than that of the previous one.

M1,M2 / M1,M2 /S / N SYSTEM (EXACT AND APPROXIMATED)

N	S	K	LAM 1	MU 1	LAM 2	MU 2	E(Q1)	VAR(Q1)	E(Q2)	VAR(Q2)	PB1	PB2
10	7	5	2.000	.500	10.000	2.000	4.18362	4.38178	2.21729	2.00416	.01866	.08010
							4.32161	3.57496	2.35169	.63479	.16080	.52966
10	7	5	2.000	1.000	10.000	2.000	2.05354	2.14995	3.23162	1.64077	.00146	.18798
							2.09890	2.33088	3.57566	1.54692	.01099	.28487
10	7	5	2.000	2.000	10.000	2.000	1.01491	1.04334	3.51095	1.54804	.00023	.26005
							1.00622	1.02747	3.57566	1.54692	.00024	.28487
10	7	5	2.000	4.000	10.000	2.000	.50348	.51040	3.56585	1.54529	.00003	.28050
							.50025	.50121	3.57566	1.54692	.00000	.28487
10	7	5	2.000	8.000	10.000	2.000	.25066	.25199	3.57438	1.54658	.00000	.28426
							.25001	.25004	3.57566	1.54692	.00000	.28487
10	7	5	5.000	.500	10.000	2.000	8.29959	2.92949	.32720	.49691	.34161	.34237
							5.57777	.57787	3.00829	1.03307	.70083	.39834
10	7	5	5.000	1.000	10.000	2.000	5.25608	4.92861	1.71643	1.78328	.06648	.09391
							5.06442	3.00945	2.35169	.63479	.26843	.52966
10	7	5	5.000	2.000	10.000	2.000	2.63556	2.83153	3.01393	1.67302	.00972	.14942
							2.69272	2.99700	3.00829	1.03307	.03079	.39834
10	7	5	5.000	4.000	10.000	2.000	1.29200	1.36999	3.46010	1.54969	.00170	.24293
							1.26530	1.31851	3.57566	1.54692	.00089	.28487
10	7	5	5.000	8.000	10.000	2.000	.63446	.65348	3.55787	1.54413	.00022	.27707
							.62573	.62841	3.57566	1.54692	.00001	.28487
10	7	5	10.000	.500	10.000	2.000	9.45577	.78782	.03414	.04312	.65211	.65212
							5.82369	.20678	3.00829	1.03307	.85003	.39834
10	7	5	10.000	1.000	10.000	2.000	8.24321	2.90247	.38518	.52641	.34950	.35009
							5.57777	.57787	3.00829	1.03307	.70083	.39834
10	7	5	10.000	2.000	10.000	2.000	5.24768	4.70528	1.77895	1.61991	.08974	.11261
							5.06442	3.00945	2.35169	.63479	.26843	.52966
10	7	5	10.000	4.000	10.000	2.000	2.68073	2.89714	3.01394	1.60858	.01976	.14967
							2.69272	2.99700	3.00829	1.03307	.03079	.39834
10	7	5	10.000	8.000	10.000	2.000	1.30951	1.41771	3.46107	1.54248	.00365	.24238
							1.26630	1.31851	3.57566	1.54692	.00089	.28487

M1,M2 / M1,M2 /S / N SYSTEM (EXACT AND APPROXIMATED)

N	S	K	LAM 1	MU 1	LAM 2	MU 2	E(Q1)	VAR(Q1)	E(Q2)	VAR(Q2)	PB1	PB2
6	4	2	2.000	.500	10.000	2.000	3.82292	2.48274	.62755	.57220	.21800	.37762
							3.58306	1.82486	.83333	.13889	.33355	.83333
6	4	2	2.000	1.000	10.000	2.000	2.12377	2.12413	1.29122	.55149	.05012	.49993
							2.11374	2.06289	1.62162	.34332	.07583	.67568
6	4	2	2.000	2.000	10.000	2.000	1.04826	1.11357	1.56138	.38311	.00965	.63317
							1.02703	1.08035	1.62162	.34332	.00676	.67568
6	4	2	2.000	4.000	10.000	2.000	.51127	.53119	1.61369	.34800	.00148	.66933
							.50263	.50955	1.62162	.34332	.00035	.67568
6	4	2	2.000	8.000	10.000	2.000	.25196	.25575	1.62072	.34382	.00017	.67492
							.25019	.25075	1.62162	.34332	.00001	.67568
6	4	2	5.000	.500	10.000	2.000	5.35359	.86327	.10204	.11707	.61273	.62250
							4.58867	.53561	.83333	.13889	.70277	.83333
6	4	2	5.000	1.000	10.000	2.000	4.29127	2.05471	.49844	.47002	.33228	.42329
							3.94722	1.43517	.83333	.13889	.43866	.83333
6	4	2	5.000	2.000	10.000	2.000	2.62744	2.31831	1.17880	.56960	.11586	.46850
							2.59585	2.22381	.83333	.13889	.13692	.83333
6	4	2	5.000	4.000	10.000	2.000	1.34326	1.42529	1.53123	.39880	.02838	.61476
							1.30016	1.37747	1.62162	.34332	.01592	.67568
6	4	2	5.000	8.000	10.000	2.000	.64948	.68910	1.60921	.35051	.00444	.66590
							.63083	.64534	1.62162	.34332	.00094	.67568
6	4	2	10.000	.500	10.000	2.000	5.74061	.31196	.01939	.02112	.80118	.80177
							4.82439	.20398	.83333	.13889	.85017	.83333
6	4	2	10.000	1.000	10.000	2.000	5.32354	.87811	.12926	.14087	.61612	.62484
							4.58867	.53561	.83333	.13889	.70277	.83333
6	4	2	10.000	2.000	10.000	2.000	4.22365	1.97698	.57865	.48809	.35289	.44020
							3.94722	1.43517	.83333	.13889	.43866	.83333
6	4	2	10.000	4.000	10.000	2.000	2.62073	2.21368	1.23532	.53070	.14366	.49126
							2.59585	2.22381	.83333	.13889	.13692	.83333
6	4	2	10.000	8.000	10.000	2.000	1.35766	1.43839	1.54512	.38777	.03723	.62325
							1.30016	1.37747	1.62162	.34332	.01592	.67568

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper presents a simple approach for the study of the following two types of multiserver finite queueing systems each with two classes of customers demanding service with different arrival and service rates: (1) Customers belonging to one class have priority over the other and the number of low priority customers that may be present in the system at any time is restricted. Customers blocked from entering service at any time are cleared from the system. (2) Customers belonging to one class are cleared when blocked

20. (Continued)

(loss system) and customers belonging to the other class are buffered (delay system) under such circumstances. Furthermore, the number of customers belonging to the loss system demanding service at any time is restricted.